

Charged Black Holes with Scalar Hair

Zhong-Ying Fan and H. Lü

Center for Advanced Quantum Studies

Department of Physics, Beijing Normal University, Beijing 100875, China

ABSTRACT

We consider a class of Einstein-Maxwell-Dilaton theories, in which the dilaton coupling to the Maxwell field is not the usual single exponential function, but one with a stationary point. The theories admit two charged black holes: one is the Reissner-Nordström (RN) black hole and the other has a varying dilaton. For a given charge, the new black hole in the extremal limit has the same $\text{AdS}_2 \times \text{Sphere}$ near-horizon geometry as the RN black hole, but it carries larger mass. We then introduce some scalar potentials and obtain exact charged AdS black holes. We also generalize the results to black p -branes with scalar hair.

Contents

1	Introduction	2
2	Charged black holes with scalar hair in $D = 4$	4
2.1	Case 1: $(a_1, a_2) = (1, -1)$	5
2.2	Case 2: $(a_1, a_2) = (\sqrt{3}, -1/\sqrt{3})$	8
2.3	The general case	9
2.4	The extremal limit and multi-centered black holes	10
2.5	Adding a scalar potential	10
3	The construction in general dimensions	11
3.1	Asymptotically-flat black holes	11
3.2	Asymptotic AdS solutions	14
4	Alternative construction and scalar hairy black p-branes	14
5	New charged AdS planar black holes	16
5.1	Class 1	17
5.2	Class 2	18
6	Conclusions	19

1 Introduction

The low-energy effective theories of superstrings, namely supergravities, provide a variety of fundamental matter fields that we can study their interactions with gravity. These include dilatonic scalars and various form fields. Under gravity, these fields can give rise to charged black holes or p -branes, which are part of fundamental constituents in the perturbative or non-perturbative string spectrum. (See, e.g. [1].)

Typically in supergravities, the dilaton ϕ couples to an n -form field strength $F_n = dA_{n-1}$ through an exponential function. The relevant Lagrangian is given by

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!}e^{a\phi}F_n^2. \quad (1)$$

where $e = \sqrt{-\det(g_{\mu\nu})}$ and a is the dilaton coupling constant. The exponential coupling function is determined by the tree-level interaction of the string. The higher-order corrections to supergravity can be extremely complicated, involving both the worldsheet α'

and the string coupling constant $g_s = \langle e^\phi \rangle$. The general structure was discussed in [2]. A concrete example is provided by $\mathcal{N} = 1$, $D = 6$ supergravity. In the canonical metric, the dilaton dependence of the kinetic energy of the Yang-Mills gauge field is [3]

$$e^{-1}\mathcal{L}_{\text{gauge}} = \sum_{\alpha} (v_{\alpha} e^{-\frac{1}{\sqrt{2}}\phi} + \tilde{v}_{\alpha} e^{\frac{1}{\sqrt{2}}\phi}) \text{tr} F_{\alpha\mu\nu} F_{\alpha}^{\mu\nu}, \quad (2)$$

where $(v_{\alpha}, \tilde{v}_{\alpha})$ are constants, with the former being essentially the Kac-Moody level [4] and the latter determined by the supersymmetry. The string one-loop correction \tilde{v}_{α} -terms play a crucial role [5] in some heterotic phase transitions [6, 7, 8].

Inspired by (2), we consider a general class of Lagrangian

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!}Z(\phi)F_n^2. \quad (3)$$

We require that the function $Z(\phi)$ become an exponential function in either the $\phi \rightarrow +\infty$ or $\phi \rightarrow -\infty$ limit. Thus we can expand Z as “ $e^{a\phi} + \dots$ ” that mimics the string loop expansion. Furthermore, we assume that $Z(\phi)$ has an stationary point, namely $dZ/d\phi = 0$ for certain ϕ_0 . There are two motivations for us to consider such a Lagrangian. The first is that the black p -branes in (1) suffer from curvature singularity on the horizon in the extremal limit, which is typically BPS or supersymmetric. In this case, the supergravity solution cannot be trusted to yield some non-perturbative information of strings. By contrast, non-dilatonic p -branes have $\text{AdS} \times \text{sphere}$ as their near-horizon geometries in such a limit. For some appropriate decoupling limit, the smooth classical supergravity background can yield informations of string theory. This leads to the famous AdS/CFT correspondence [9].

The dilaton coupling function $Z(\phi)$ for (2) has a stationary point. Thus the dilaton can be decoupled. This implies that the associated p -brane can have smooth horizon even in the extremal limit. It is natural to assume that the singularity associated with the extremal limit of the p -branes in (1) is an artifact of the low-energy effect action of the lowest order. Loop correction is expected to smooth out the singularity and extremal p -branes in strings may also have non-vanishing entropy [10]. The general loop-corrected actions of strings are very complicated to construct and few examples are known. The Lagrangian (3) can be viewed as a toy model for such string loop effects.

The second motivation is to do with uniqueness of black holes. The standard no-hair theorem states that a black hole is completely specified by the mass, charge and angular momenta. Recently, many examples of scalar hairy black holes have been constructed [11–26], and their general first law of thermodynamics were derived [27, 28]. In supergravities, charged black holes also generally involve scalar fields and yet the uniqueness theorem

appears to continue to hold for the large number of such solutions constructed in literature. It is of interest to construct a simple counter example. It is clear that if $Z(\phi)$ in (3) has a stationary point as in (2), then the usual Reissner-Nordström (RN) black hole with the dilaton decoupled must be a solution. If we can construct a further different black hole with the same mass and charge, but non-vanishing dilaton, the uniqueness theorem is then broken. The Lagrangian (3) can thus stand alone as an example for studying non-uniqueness of charged black holes. Numerically it can be established that charged black holes for a generic $Z(\phi)$ may indeed exist. In this paper, we find non-trivial exact solutions when Z is given by

$$Z^{-1} = \cos^2 \omega e^{a_1 \phi} + \sin^2 \omega e^{a_2 \phi}, \quad \text{with} \quad a_1 a_2 = -\frac{2(n-1)(D-n-1)}{D-2}. \quad (4)$$

Here ω is some coupling constant. For $\omega = 0$ or $\frac{1}{2}\pi$, the function Z becomes an exponential function of ϕ . In general Z has a stationary point for which $dZ/d\phi$ vanishes. This dilaton coupling to the field strength resembles those in the STU-model truncation [29] of $D = 4$ ω -deformed gauged supergravity [30, 31].

The paper is organized as follows. In section 2, we present new charged black holes in four dimensions. We focus on the examples that are associated with supergravities. We discuss their global properties and obtain the first law of thermodynamics. We compare the new black holes with the RN black hole. We also obtain the multi-center extremal solutions. We then introduce some scalar potentials and obtain charged AdS black holes. In section 3, give a construction of the new black holes for general dimensions. In section 4, we give an alternative construction that allows us to construct p -branes with scalar hair in general dimensions. In section 5, we construct further new charged AdS planar black holes. We conclude the paper in section 6.

2 Charged black holes with scalar hair in $D = 4$

In this section, we consider Einstein-Maxwell-Dilaton theory in four dimensions, corresponding to $D = 4$ and $n = 2$. The condition for (a_1, a_2) in (4) becomes $a_1 a_2 = -1$. In supergravities with an exponential dilaton coupling, a_1 may take values $\pm(\sqrt{3}, 1, 1/\sqrt{3}, 0)$. We thus first consider the two special cases, namely $(1, -1)$ and $(\sqrt{3}, -1/\sqrt{3})$, in greater detail and summarize the general results later in this section.

2.1 Case 1: $(a_1, a_2) = (1, -1)$

We start the discussion by considering an example of charged black holes with scalar hair in four dimensions. The theory consists of the metric, a dilaton ϕ and a Maxwell field A . The Lagrangian is given by

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}ZF^2, \quad Z^{-1} = \cos^2\omega e^\phi + \sin^2\omega e^{-\phi}. \quad (5)$$

where $F = dA$. Naively, the parameter ω has a range of $\frac{1}{2}\pi$; however, owing to the symmetry of $\omega \rightarrow \omega + \frac{1}{4}\pi$ and $\phi \rightarrow -\phi$, ω has a range of $\frac{1}{4}\pi$ instead. For latter purpose we choose the range of ω lies in $[\frac{1}{4}\pi, \frac{1}{2}\pi]$. When $\omega = \frac{1}{2}\pi$, Z^{-1} then involves only one exponential term and the Lagrangian is invariant under the constant shift of the dilaton provided that it is compensated by an appropriate scaling of the gauge potential A . For generic fixed ω , however, such a freedom is deprived. The $\omega = \frac{1}{2}\pi$ Lagrangian can be embedded in $D = 4$, $\mathcal{N} = 4$ supergravity.

To construct black hole solutions to (5), we first note that $Z(\phi)$ has a stationary point for $\omega \neq \frac{1}{2}\pi$, namely

$$\left. \frac{dZ}{d\phi} \right|_{\phi=\phi_0} = 0, \quad \text{for} \quad e^{\phi_0} = \tan\omega. \quad (6)$$

Thus the dilaton can be decoupled, giving rise to the Einstein-Maxwell theory

$$e^{-1}\mathcal{L} = R - \frac{1}{4\sin(2\omega)}F^2. \quad (7)$$

The Lagrangian admits the well-known RN black hole

$$\begin{aligned} ds_4^2 &= -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega_2^2, \quad A = \frac{4\sin(2\omega) Q_e}{r} dt, \\ f &= 1 - \frac{2M}{r} + \frac{4\sin(2\omega) Q_e^2}{r^2}. \end{aligned} \quad (8)$$

where the two integration constants M and Q_e are the mass and electric charge respectively. For the solution to be a black hole, the mass and charge must satisfy the inequality

$$M \geq M_{\text{RN}}^{\text{ext}} \equiv 2\sqrt{\sin(2\omega)} Q_e. \quad (9)$$

The solution becomes extremal when the above inequality is saturated and the black hole has zero temperature with the $\text{AdS}_2 \times S^2$ near-horizon geometry.

In addition to the RN black hole, we find that the theory (5) admits a different charged black hole in which the dilaton is not a constant. We shall give the detail construction later. For now, we simply present the solution

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r(r+q)d\Omega_2^2, \quad f = 1 + \frac{Q^2(q + (2r+q)\cos(2\omega))}{4qr(r+q)},$$

$$e^\phi = 1 + \frac{q}{r}, \quad A = \frac{Q(r + q \cos^2 \omega)}{r(r + q)} dt, \quad (10)$$

The solution involves two integration constant q and Q , which, as we shall see presently, parameterize the mass and electric charge of the solution. We assume that $q > 0$. With this choice, the spacetime has no curvature singularity in the $r > 0$ region. For the solution to describe a black hole, the curvature singularity at $r = 0$ has to be shielded by an event horizon at certain $r = r_0 > 0$; the curvature singularity at $r = -q < 0$ is irrelevant for our discussion.

In the large r expansion, we have

$$-g_{tt} = 1 + \frac{Q^2 \cos(2\omega)}{2qR} + \frac{Q^2}{4R^2} + \dots, \quad (11)$$

where $R = \sqrt{r(r + q)}$ is radius of the foliating 2-sphere. We can read off the ADM mass

$$M = -\frac{1}{4} \cos(2\omega) \frac{Q^2}{q}. \quad (12)$$

We see now the reason that we have chosen $\omega \in [\frac{1}{4}\pi, \frac{1}{2}\pi]$. (Of course, one can perfectly choose $\omega \in [0, \frac{1}{4}\pi]$, in which case, we can simply let q be negative and the curvature singularity $r = -q > 0$ becomes the relevant one to discuss.) The electric charge of the solution is given by

$$Q_e = \frac{1}{16\pi} \int_{r \rightarrow \infty} Z_* F = \frac{1}{4} Q. \quad (13)$$

To determine the relation of the mass M and Q_e so that the solution describe a black hole, we note that the function f approaches $+\infty$ at $r = 0$ and becomes 1 as $r \rightarrow +\infty$. It follows from (11) that for positive M , there must be a minimum for f at some $r = r_{\min} > 0$. It is straightforward to determine that

$$r_{\min} = \frac{q \cos \omega}{\sin \omega - \cos \omega}, \quad f^{\min} = f(r_{\min}) = \frac{1}{2q^2} (Q^2 \sin(2\omega) - Q^2 + 2q^2). \quad (14)$$

Thus the solution describes a black hole provided that $f^{\min} \leq 0$, i.e.

$$\sin(2\omega) \leq 1 - \frac{2q^2}{Q^2}. \quad (15)$$

When the inequality is satisfied, f has two real positive roots

$$r_{\pm} = \frac{1}{4q} \left(\pm \sqrt{(Q^2 - 2q^2)^2 - Q^4 \sin^2(2\omega) - 2q^2 - Q^2 \cos(2\omega)} \right). \quad (16)$$

Using the standard technique, we obtain the temperature and the entropy

$$T = \frac{2(r_0 + q)^2 \cos^2 \omega - 2r_0^2 \sin^2 \omega}{4\pi r_0(r_0 + q)(q + (2r_0 + q) \cos(2\omega))}, \quad S = r_0(r_0 + q)\pi. \quad (17)$$

where $r_0 = r_+$. The electric potential difference between the horizon and the asymptotic infinity is

$$\Phi_e = \frac{Q(r_0 + q \cos^2 \omega)}{r_0(r_0 + q)}. \quad (18)$$

It is then straightforward to verify that the first law of black hole thermodynamics

$$dM = TdS + \Phi_e dQ_e \quad (19)$$

is satisfied. These quantities satisfy also the Smarr relation

$$M = 2TS + \Phi_e Q_e. \quad (20)$$

Thus we see that for given mass M and electric charge Q_e , the theory (5) admits two black holes, one is the RN black hole with constant ϕ , whilst the other is the scalar hairy one involving a varying ϕ .

The new scalar hairy solution also has an extremal limit in which T vanishes, corresponding to

$$\tan \omega = \frac{r_0 + q}{r_0} = e^{\phi} \Big|_{r=r_0}. \quad (21)$$

Thus we see that $\phi|_{r=r_0} = \phi_0$, where ϕ_0 was defined in (6). The mass of the extremal black hole with scalar hair is then given by

$$M_{\text{SH}}^{\text{ext}} = \sqrt{2}(\cos \omega + \sin \omega)Q_e. \quad (22)$$

It is easy to verify that for the same electric charge, the mass of the extremal hairy black hole is in general bigger than the extremal RN one, i.e.

$$M_{\text{SH}}^{\text{ext}} \geq M_{\text{RN}}^{\text{ext}}. \quad (23)$$

In particular, this implies that in the extremal limit, both the new and RN black holes with the same electric charge have the same $\text{AdS}_2 \times S^2$ structure as their near horizon geometry, but the asymptotic behaviors are different; the new solution having a larger mass. The inequality (23) is saturated for $\omega = \frac{1}{4}\pi$ for which the scalar hairy black hole degenerates to become the RN black hole.

Finally, we note that both the new and the RN black holes have two horizons, inner and outer horizon. The product of the entropies of the two horizons are quantized in the same way

$$S_+ S_- = 16\pi^2 Q_e^4 \sin^2(2\omega). \quad (24)$$

The universality of this property of two very different solutions of the theory is rather telling that there may be a same conformal field theory that is responsible for the interpretation

of the black hole entropy. (See, e.g. [34, 35].) This entropy product rule also implies that although for given electric charge Q_e , the mass of RN and the new scalar hairy black hole is different, but their entropy is the same in the extremal limit.

2.2 Case 2: $(a_1, a_2) = (\sqrt{3}, -1/\sqrt{3})$

The Lagrangian for the second example takes the same form as (5), but now with Z given by

$$Z^{-1} = \cos^2 \omega e^{\sqrt{3}\phi} + \sin^2 \omega e^{-\frac{1}{\sqrt{3}}\phi}. \quad (25)$$

The range of ω lies in the interval $[0, \frac{1}{2}\pi]$. We pick up this example to discuss in some detail because the theory can be embedded in supergravity when $\omega = 0$ or $\frac{1}{2}\pi$. The function Z has a stationary point $\phi = \phi_0$ with

$$e^{\frac{4}{\sqrt{3}}\phi_0} = \frac{1}{3} \tan^2 \omega. \quad (26)$$

Thus the dilaton can be decoupled, giving rise to Einstein Maxwell theory

$$e^{-1}\mathcal{L} = R - \frac{1}{4\Theta^2}F^2, \quad (27)$$

where

$$\Theta = 2 \cdot 3^{-\frac{3}{8}} (\cos \omega)^{\frac{1}{4}} (\sin \omega)^{\frac{3}{4}}. \quad (28)$$

Thus the theory admits the RN black hole and its mass and charge satisfy

$$M \geq M_{\text{RN}}^{\text{ext}} = 2\Theta Q_e. \quad (29)$$

The bound is saturated in the extremal limit.

We find that the charged scalar hairy black hole is given by

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 \left(1 + \frac{q}{r}\right)^{\frac{3}{2}} d\Omega_2^2, \quad A = \frac{Q(r + q \cos^2 \omega)}{r(r + q)},$$

$$e^{\frac{2}{\sqrt{3}}\phi} = 1 + \frac{q}{r}, \quad f = \frac{1}{\sqrt{r(r + q)^3}} \left(r(r + q) + \frac{Q^2}{3q} (3(r + q) \cos^2 \omega - r \sin^2 \omega) \right). \quad (30)$$

Following the same technique in the earlier discussion, we find that the mass and the electric charge of the black hole is given by

$$M = \frac{1}{12q} (3q^2 + 2Q^2 - 8Q^2 \cos^2 \omega), \quad Q_e = \frac{1}{4}Q. \quad (31)$$

It is easy to establish that when the parameters satisfy

$$(2Q \cos \omega - \frac{3}{2}q)^2 \geq \frac{3}{4}q^2 - Q^2, \quad (32)$$

the function $f(r)$ has two positive roots $r_{\pm} > 0$. The horizon is located at $r_0 \equiv r_+ \geq r_-$. The temperature, the entropy, and electric potential are

$$\begin{aligned} T &= \frac{3(r_0 + q)^2 \cos^2 \omega - r_0^2 \sin^2 \omega}{4\pi \sqrt{r_0(r_0 + q)^3} (3(r_0 + q) \cos^2 \omega - r_0 \sin^2 \omega)}, \\ S &= \pi \sqrt{r_0(r_0 + q)^3}, \quad \Phi_e = \frac{Q(r_0 + q \cos^2 \omega)}{r_0(r_0 + q)}. \end{aligned} \quad (33)$$

We verify that both the first law (19) and the Smarr relation (20) are satisfied. In the extremal limit, where the temperature vanishes, the mass is proportional to the electric charge

$$M_{\text{SH}}^{\text{ext}} = (\cos \omega + \sqrt{3} \sin \omega) Q_e. \quad (34)$$

The condition (32) for the existence of event horizon is equivalent to $M \geq M_{\text{SH}}^{\text{ext}}$. In general $M_{\text{SH}}^{\text{ext}} \geq M_{\text{RN}}^{\text{ext}}$ with the equality is achieved at $\omega = \frac{1}{3}\pi$, in which case, the hairy black hole degenerates to the RN black hole.

2.3 The general case

We now consider the general case with $a_1 a_2 = -1$. We reparameterize the constants as

$$a_1 = \sqrt{\frac{1-\mu}{1+\mu}}, \quad a_2 = -\sqrt{\frac{1+\mu}{1-\mu}}. \quad (35)$$

The dilaton coupling function z is thus given by

$$Z^{-1} = \cos^2 \omega e^{\sqrt{\frac{1-\mu}{1+\mu}} \phi} + \sin^2 \omega e^{-\sqrt{\frac{1+\mu}{1-\mu}} \phi}, \quad (36)$$

It has a stationary point, given by

$$e^{\frac{1}{\sqrt{1-\mu^2}} \phi_0} = \sqrt{\frac{1+\mu}{1-\mu}} \tan \omega. \quad (37)$$

The reduced Einstein-Maxwell theory (27) has

$$\Theta = \frac{\sin(2\omega)}{\sqrt{1-\mu^2}} \left(\sqrt{\frac{1+\mu}{1-\mu}} \cot \omega \right)^\mu. \quad (38)$$

We find that the scalar hairy black hole is given by

$$\begin{aligned} ds^2 &= -f dt^2 + \frac{dr^2}{f} + r^{1+\mu} (r+q)^{1-\mu} d\Omega_2^2, \quad A = \frac{Q(r+q \cos^2 \omega)}{r(r+q)}, \\ e^{\frac{1}{\sqrt{1-\mu^2}} \phi} &= 1 + \frac{q}{r}, \quad f = \left(1 + \frac{q}{r}\right)^\mu \left(1 + \frac{\cos^2 \omega Q^2}{2(1+\mu)qr} - \frac{\sin^2 \omega Q^2}{2(1-\mu)q(r+q)}\right). \end{aligned} \quad (39)$$

The mass and electric charge are given by

$$M = \frac{Q^2 \sin^2 \omega}{4(1-\mu)q} - \frac{Q^2 \cos^2 \omega}{4(1+\mu)q} - \frac{1}{2} \mu q, \quad Q_e = \frac{1}{4} Q. \quad (40)$$

For appropriate parameter regions, the event horizon exists and the first law (19) and smarr relation (20) can be shown to be satisfied. For a given electric charge Q_e , the black hole mass must satisfy the inequality

$$M \geq M_{\text{SH}}^{\text{ext}} \equiv \sqrt{2}(\sqrt{1+\mu} \cos \omega + \sqrt{1-\mu} \sin \omega) Q_e. \quad (41)$$

When the equality is achieved, the black hole becomes extremal with zero temperature and the corresponding near horizon geometry is $\text{AdS}_2 \times S^2$. For the extremal solution, the dilaton runs from $\phi = \phi_0$ on the horizon to $\phi = 0$ to asymptotic flat infinity. For a given electric charge Q_e , the mass of extremal solution of scalar hairy black hole is in general bigger than that of the RN black hole, namely

$$M_{\text{SH}}^{\text{ext}} \geq M_{\text{RN}}^{\text{ext}}. \quad (42)$$

The equality occurs when $\tan \omega = \sqrt{1-\mu}/\sqrt{1+\mu}$ in which case, the scalar hairy black hole degenerates to the RN black hole.

The general black holes have two horizons, one is outer and the other is inner. The product of the two horizon entropies is quantized:

$$S_+ S_- = 64\pi^2 \left(\frac{\cos^2 \omega}{1+\mu}\right)^{1+\mu} \left(\frac{\sin^2 \omega}{1-\mu}\right)^{1-\mu} Q_e^4. \quad (43)$$

2.4 The extremal limit and multi-centered black holes

As in the case of RN black holes, in the extremal limit, the electric force and gravity balance out and one can have multi-centered solutions. We find that the multi-centered solutions for the general Z (36) are given by

$$\begin{aligned} ds_4^2 &= H_1^{-1+\mu} H_2^{-1+\mu} dt^2 + H_1^{1+\mu} H_2^{1-\mu} dx^i dx^i, \\ A &= \left(\frac{\sqrt{2(1+\mu)} \cos \omega}{H_1} + \frac{\sqrt{2(1-\mu)} \sin \omega}{H_2} \right) dt, \quad \phi = \sqrt{1-\mu^2} \log \frac{H_2}{H_1}, \\ H_1 &= 1 + \frac{\cos \omega}{\sqrt{2(1-\mu)}} H_0, \quad H_2 = 1 + \frac{\sin \omega}{\sqrt{2(1+\mu)}} H_0, \end{aligned} \quad (44)$$

where H_0 is given by

$$H_0 = \sum_i \frac{Q_i}{|\vec{x} - \vec{x}_i|}. \quad (45)$$

The solution reduces to $\text{AdS}_2 \times S^2$ near the location of each charge, namely $\vec{x} \rightarrow \vec{x}_i$.

2.5 Adding a scalar potential

The charged black hole solutions we considered are all asymptotic to the flat Minkowski spacetime. We would like to add a scalar potential to the Lagrangian and construct charged black holes that are asymptotic to AdS.

For $\omega = 0$ or $\omega = \frac{1}{2}\pi$, charged AdS black holes were previously constructed. The scalar potential is given by

$$V(\phi) = -(g^2 - \alpha)V_0(\phi) - \alpha V_0(-\phi), \quad (46)$$

with

$$V_0(\phi) = (1 + \mu)(1 + 2\mu)e^{-\sqrt{\frac{1-\mu}{1+\mu}}\phi} + (1 - \mu)(1 - 2\mu)e^{\sqrt{\frac{1+\mu}{1-\mu}}\phi} + 4(1 - \mu^2)e^{\frac{\mu}{\sqrt{1-\mu^2}}\phi}. \quad (47)$$

We find that the Lagrangian admits charged black hole solution for general ω . The solution is given by

$$\begin{aligned} ds^2 &= -f dt^2 + \frac{dr^2}{f} + r^{1+\mu}(r+q)^{1-\mu} d\Omega_{2,k}^2, \\ A &= \frac{Q(r+q \cos^2 \omega)}{r(r+q)}, \quad e^{\frac{1}{\sqrt{1-\mu^2}}\phi} = 1 + \frac{q}{r}, \end{aligned} \quad (48)$$

with

$$\begin{aligned} f &= (g^2 - \alpha)\left(1 + \frac{q}{r}\right)^\mu (r(r+q) - \mu q(2r+q) + 2\mu^2 q^2) + \alpha r^2 \left(1 + \frac{q}{r}\right)^{1-\mu} \\ &\quad + \left(1 + \frac{q}{r}\right)^\mu \left(k + \frac{\cos^2 \omega Q^2}{2(1+\mu)qr} - \frac{\sin^2 \omega Q^2}{2(1-\mu)q(r+q)}\right). \end{aligned} \quad (49)$$

Here k takes the values 1, 0 or -1 corresponding to the unit round sphere, torus or hyperbolic space for $d\Omega_{2,k}^2$.

It is worth commenting that although both V and Z have stationary points, they do not coincide. Thus the dilaton cannot be decoupled and the RN-AdS black hole is not a solution of the theory.

3 The construction in general dimensions

In the previous section, we presented a class of charged black holes with scalar hair in four dimensions. In this section, we show the method of construction. We give the construction in general dimensions, but continue to focus on black holes. We shall present an alternative construction in the next section where we obtain a class of black p -branes with scalar hair.

3.1 Asymptotically-flat black holes

We now consider Einstein gravity coupled to a Maxwell field with a generic dilaton coupling in general D dimensions

$$e^{-1}\mathcal{L}_D = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}Z(\phi)F^2, \quad (50)$$

The equations of motion are

$$\begin{aligned}\square\phi &= \frac{1}{4}\frac{dZ}{d\phi}F^2, & \nabla_\mu(ZF^{\mu\nu}) &= 0, \\ E_{\mu\nu} &\equiv R_{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}Z(F_{\mu\nu}^2 - \frac{1}{2(D-2)}F^2g_{\mu\nu}) &= 0,\end{aligned}\quad (51)$$

We consider the ansatz

$$ds^2 = -H^{-1}f dt^2 + H^{\frac{1}{D-3}}\left(\frac{dr^2}{f} + r^2 d\Omega_{D-2,k}^2\right), \quad A = \psi dt. \quad (52)$$

The functions H, f, ψ and the dilaton ϕ are all functions of r only. The metric $d\Omega_{D-2,k}^2$ with $k = 1, 0, -1$ denotes that of the maximally symmetric space with $R_{ij} = (D-3)k\delta_{ij}$. In other words, it is for round sphere, torus, or hyperbolic $(D-2)$ -space respectively. For asymptotic flat black holes, we must have $k = 1$, but we keep k generic for discussing local solutions.

The equation of motion of A implies that

$$\psi' = \frac{Q}{ZH r^{D-2}}, \quad (53)$$

where Q is an integration constant related to the electric charge

$$Q_e = \frac{1}{16\pi} \int_{r \rightarrow \infty} Z * F = \frac{Q \Sigma_{D-2}}{16\pi}, \quad (54)$$

where Σ_{D-2} is the volume of the space associated with $d\Omega_{D-2,k}^2$. This is the most general ansatz that respects the isometry and it was first considered in [18]. A nice property of this ansatz is that the combination of the Einstein equations $E_t{}^t - (D-3)E_i{}^i$ yields

$$r^2 f'' + (3D-8)rf' + 2(D-3)^2(f-k) = 0. \quad (55)$$

Thus we have

$$f = k + \frac{c_1}{r^{D-3}} + \frac{c_2}{r^{2(D-3)}}, \quad (56)$$

where c_1, c_2 are two integration constants. The combination $E_t{}^t - E_r{}^r$ gives

$$\frac{2(D-3)}{D-2}\phi'^2 + \frac{2H''}{H} - \frac{H'^2}{H^2} + \frac{2(D-2)H'}{rH} = 0. \quad (57)$$

A further ansatz is thus necessary for obtaining exact solutions. We follow [18] and assume the dilaton is given by

$$\phi = \sqrt{\frac{D-2}{2(D-3)}}\sqrt{1-\mu^2}\log\frac{H_1}{H_2}, \quad H_i = 1 + \frac{q_i}{r^{D-3}}, \quad i = 1, 2. \quad (58)$$

Then the function H can be solved exactly, namely [18]

$$H = H_1^{1+\mu} H_2^{1+\mu} (cH_1^{-\mu} + (1-c)H_2^{-\mu})^2, \quad (59)$$

where c is an integration constant. (Another integration constant is overall scale of H which we fix so that H becomes unity as $r \rightarrow \infty$.) We can then read off the function Z as a function r with the remainder of the Einstein equations. Using (58) we can rewrite Z in terms of ϕ . We would like to insist that $Z(\phi)$ is independent of the parameters q and Q , this implies that we must have $c = 0$ or $c = 1$ for generic k . (New possibility arises for $k = 0$, which we shall discuss in section 5.) The choice of $c = 0$ and $c = 1$ is simply an interchange q_1 and q_2 and hence we shall choose without loss of generality $c = 0$, and hence

$$H = H_1^{1+\mu} H_2^{1-\mu}. \quad (60)$$

The function $Z(\phi)$ is then given by

$$\begin{aligned} Z^{-1} &= \cos^2 \omega e^{a_1 \phi} + \sin^2 \omega e^{a_2 \phi}, \\ a_1 &= \sqrt{\frac{2(D-3)}{D-2}} \sqrt{\frac{1+\mu}{1-\mu}}, \quad a_2 = -\sqrt{\frac{2(D-3)}{D-2}} \sqrt{\frac{1-\mu}{1+\mu}}. \end{aligned} \quad (61)$$

Note that we have chosen the constants c_1 and c_2 in (56) such that the parameters q_i and Q does not appear in $Z(\phi)$ explicitly. The metric function f is now given by

$$f = k H_1 H_2 + \frac{Q^2}{(D-2)(D-3)(q_1 - q_2)r^{D-3}} \left(\frac{\cos^2 \omega}{1-\mu} H_1 - \frac{\sin^2 \omega}{1+\mu} H_2 \right). \quad (62)$$

Thus the theory and the corresponding black hole solution are both fully determined.

The function Z has a stationary point and hence the dilaton can be decoupled, giving rise to Einstein-Maxwell theory (27) with

$$\Theta = 2 \left(\frac{\cos^2 \omega}{1-\mu} \right)^{\frac{1}{2}(1-\mu)} \left(\frac{\sin^2 \omega}{1+\mu} \right)^{\frac{1}{2}(1+\mu)}. \quad (63)$$

This quantity turns out to be dimensionally independent. At large r , we find that

$$g_{tt} = -k + \left(k\mu q - \frac{Q^2(\mu + \cos(2\omega))}{(D-2)(D-3)(1-\mu^2)q} \right) \frac{1}{R^{D-3}} - \frac{Q^2}{2(D-2)(D-3)R^{2(D-3)}} + \dots, \quad (64)$$

where $q = q_1 - q_2$ and $R = r H^{\frac{1}{2(D-3)}}$. Thus the ADM mass is given by

$$M = \frac{\Sigma}{16\pi} \left((D-2)k\mu q - \frac{Q^2(\mu + \cos(2\omega))}{(D-3)(1-\mu^2)q} \right). \quad (65)$$

Here it is understood that $k = 1$. The Maxwell field is given by

$$A = -\frac{Q}{D-3} \left(\frac{\cos^2 \omega}{r^{D-3} + q_2} + \frac{\sin^2 \omega}{r^{D-3} + q_1} \right) dt. \quad (66)$$

It follows that the electric charges and potential are given by

$$Q_e = \frac{Q}{16\pi} \Sigma, \quad \Phi_e = \frac{Q}{D-3} \left(\frac{\cos^2 \omega}{r_0^{D-3} + q_2} + \frac{\sin^2 \omega}{r_0^{D-3} + q_1} \right), \quad (67)$$

where r_0 is the location of the horizon. With the standard technique for calculating the temperature and entropy T, S , we can easily verify the first law of thermodynamics and the Smarr relation, given by

$$dM = TdS + \Phi_e dQ_e, \quad M = \frac{D-2}{D-3}TS + \Phi_e Q_e. \quad (68)$$

The general solution also has two horizons with the product of the two corresponding entropies being quantized, namely

$$S_+ S_- = \frac{1}{16} \Sigma^2 \left(\frac{Q^2}{(D-2)(D-3)} \left(\frac{\cos^2 \omega}{1-\mu} \right)^{1-\mu} \left(\frac{\sin^2 \omega}{1+\mu} \right)^{1+\mu} \right)^{\frac{D-2}{D-3}}. \quad (69)$$

3.2 Asymptotic AdS solutions

We now consider adding a scalar potential V to the Lagrangian. For the metric ansatz (52), the results were already obtained in [18]. The scalar potential turns out to be

$$\begin{aligned} V = & -\frac{1}{2}(D-2)g^2 e^{\frac{\mu-1}{\nu}\Phi} \left[(\mu-1)((D-2)\mu-1)e^{\frac{2}{\nu}\Phi} - 2(D-2)(\mu^2-1)e^{\frac{1}{\nu}\Phi} \right. \\ & \left. + (\mu+1)((D-2)\mu+1) \right] \\ & - \frac{(D-3)^2}{2(3D-7)} (\mu+1) \alpha e^{-\frac{1}{\nu}(4+\frac{\mu+1}{D-3})\Phi} (e^{\frac{1}{\nu}\Phi} - 1)^{3+\frac{2}{D-3}} \\ & \times \left[(3D-7)e^{\frac{1}{\nu}\Phi} {}_2F_1\left[2, 1 + \frac{(D-2)(\mu+1)}{D-3}; 3 + \frac{2}{D-2}; 1 - e^{\frac{1}{\nu}\Phi}\right] + \right. \\ & \left. - ((3D-7) + (D-2)(\mu-1)) {}_2F_1\left[3, 2 + \frac{(D-2)(\mu+1)}{D-3}; 4 + \frac{2}{D-2}; 1 - e^{\frac{1}{\nu}\Phi}\right] \right], \quad (70) \end{aligned}$$

where $\nu = \sqrt{1-\mu^2}$ and

$$\Phi = \sqrt{\frac{2(D-3)}{D-2}} \phi. \quad (71)$$

The solution is then given by

$$\begin{aligned} f \rightarrow & f + g^2 r^2 \left(H_1^{1+\mu} H_2^{1-\mu} \right)^{\frac{D-2}{D-3}} \\ & - \alpha r^2 H_2 (H_1 - H_2)^{\frac{D-1}{D-3}} {}_2F_1\left[1, \frac{D-2}{D-3}(1+\mu); \frac{2(D-2)}{D-3}; 1 - \frac{H_2}{H_1}\right]. \quad (72) \end{aligned}$$

Since now the solutions are asymptotic AdS, we can let $k = 1, 0, -1$ instead of only $k = 1$ in the asymptotic flat case.

4 Alternative construction and scalar hairy black p -branes

In the previous section, we construct a class of charged scalar hairy black holes that are asymptotic to flat spacetimes. The Z function is given by (61) with

$$a_1 a_2 = -\frac{2(D-3)}{D-2}. \quad (73)$$

This is precisely a condition that one can construct analytical two-charge black holes for the Lagrangian [32]:

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{a_1\phi}(F^1)^2 - \frac{1}{4}e^{a_2\phi}(F^2)^2. \quad (74)$$

This suggests that a relation exists between our solution and the two-charge solutions constructed in [32].

To establish an even more general relation, we consider a Lagrangian involving two $\tilde{n} = D - n$ form field strengths $F_{\tilde{n}}^i = dA_{\tilde{n}-1}^i$ in D -dimensions

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2\tilde{n}!}e^{a_1\phi}(F_{\tilde{n}}^1)^2 - \frac{1}{2\tilde{n}!}e^{a_2\phi}(F_{\tilde{n}}^2)^2, \quad (75)$$

The dilaton coupling constants (a_1, a_2) satisfy

$$a_1 a_2 = -\frac{2(n-1)(D-n-1)}{D-2}. \quad (76)$$

We can parameterize the a_1 and a_2 as following

$$a_1^2 = \frac{4}{N_1} + a_1 a_2, \quad a_2^2 = \frac{4}{N_2} + a_1 a_2, \quad (77)$$

In this parametrization, we have the following two identities

$$a_1 N_1 + a_2 N_2 = 0, \quad a_1 a_2 (N_1 + N_2) = -4. \quad (78)$$

The Lagrangian admits the $(n-2)$ -brane with magnetic charges. The metric and the dilaton are given by (see e.g. [33, 36, 37],)

$$\begin{aligned} ds^2 &= (H_1^{N_1} H_2^{N_2})^{-\frac{D-n-1}{D-2}} (-f dt^2 + dx^i dx^i) + (H_1^{N_1} H_2^{N_2})^{\frac{n-1}{D-2}} \left(\frac{dr^2}{f} + r^2 d\Omega_{D-n}^2 \right), \\ \phi &= \frac{1}{2} a_1 N_1 \log H_1 + \frac{1}{2} a_2 N_2 \log H_2, \\ f &= 1 - \frac{\mu}{r^{D-n-1}}, \quad H_i = 1 + \frac{\mu \sinh \delta_i^2}{r^{D-n-1}}, \quad i = 1, 2. \end{aligned} \quad (79)$$

The gauge fields are given by their field strengths

$$F_{\tilde{n}}^i = P_i \Omega_{D-n}, \quad P_i = \frac{1}{2} (D - n - 1) \mu \sqrt{N_i} \sinh(2\delta_i). \quad (80)$$

The solutions have three integration constants $(\mu, \delta_1, \delta_2)$, giving rise to the mass and two magnetic charges. We now redefine the parameters to be

$$P_1 = \cos^2 \omega Q, \quad P_2 = \sin^2 \omega Q, \quad (81)$$

The corresponding solution can be also obtained from the Lagrangian

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2\tilde{n}!}(\cos^2 \omega e^{a_1\phi} + \sin^2 \omega e^{a_2\phi})(F_{\tilde{n}}^n)^2. \quad (82)$$

The corresponding field strength is given by

$$F_{\tilde{n}} = Q \Omega_{D-2}. \quad (83)$$

We perform electric and magnetic duality on the \tilde{n} -form field strength to become an n -form $F_n = dA_{n-1}$. The corresponding theory becomes

$$e^{-1}\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!}Z(\phi)F_n^2, \quad Z^{-1} = \cos^2\omega e^{a_1\phi} + \sin^2\omega e^{a_2\phi}. \quad (84)$$

The electrically charged black p -brane solution is then given by (79), but with the field gauge potential

$$A_{n-1} = a dt \wedge dx^1 \wedge \cdots \wedge dx^{n-2}, \quad a' = \frac{Q}{Z r^{D-n-2}} (H_1^{N_1} H_2^{N_2})^{\frac{1}{2}a_1 a_2}. \quad (85)$$

The solution now involves two integration constants (μ, Q) , giving rise to the mass and electric charge. The parameter ω is now a coupling constant in the theory. It should be pointed out again that the theory admits also a non-dilatonic p -brane carrying the same mass and charge where the dilaton is a constant, as in the case of the black hole examples discussed earlier.

The multi-center $(n-1)$ -brane solution in the extremal limit can also be easily obtained, given by

$$\begin{aligned} ds^2 &= (H_1^{N_1} H_2^{N_2})^{-\frac{D-n-1}{D-2}} dx^\mu dx^\nu \eta_{\mu\nu} + (H_1^{N_1} H_2^{N_2})^{\frac{n-1}{D-2}} dy^m dy^m, \\ \phi &= \frac{1}{2}a_1 N_1 \log H_1 + \frac{1}{2}a_2 N_2 \log H_2, \\ A_n &= \left(\frac{\sqrt{N_1} \cos \omega}{H_1} + \frac{\sqrt{N_2} \sin \omega}{H_2} \right) dt \wedge dx^1 \cdots \wedge dx^{n-1}, \\ H_1 &= 1 + \frac{\cos \omega}{\sqrt{N_1}} H_0, \quad H_2 = 1 + \frac{\sin \omega}{\sqrt{N_2}} H_0, \end{aligned} \quad (86)$$

where

$$H_0 = \sum_{\alpha} \frac{Q_{\alpha}}{|\vec{y} - \vec{y}_{\alpha}|^{D-n-1}}. \quad (87)$$

The geometry around each center of the charges is $\text{AdS}_n \times S^{D-n}$.

5 New charged AdS planar black holes

As was discussed under (59) in section 3, we could construct more possibilities of the Z function when the spatial section is flat, i.e. $k = 0$. Although for a black hole to be asymptotic flat, we must have $k = 1$, the $k = 0$ case is allowed for asymptotic AdS solutions. In this section, we consider again the EMD theory with general coupling function Z and scalar potential V .

5.1 Class 1

The class-1 solutions are specified by the metric ansatz

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^{1+\mu}(r+q)^{1-\mu} dx^i dx^i. \quad (88)$$

Large classes of neutral scalar hairy AdS planar black holes were obtained in [17, 26]. The scalar potential is

$$\begin{aligned} V(\phi) = & \frac{1}{4}(D-2)g^2 e^{-\lambda\mu\phi} \left(2D(\mu^2 - 1) + (2-D+D\mu)(1-\mu)e^{-\lambda\phi} \right. \\ & + (2-D-D\mu)(1+\mu)e^{\lambda\phi} \Big) - \frac{1}{4}(D-2)\alpha e^{-\lambda\phi} (e^{\lambda\phi} - 1)^{D-1} \times \\ & \left(2(D-1)(1-\mu + (1+\mu)e^{\lambda\phi}) e^{-\frac{\phi}{\lambda(1+\mu)}} \right. \\ & - e^{-\lambda\mu\phi} \left[(D-1)(1-\mu + (1+\mu)e^{\lambda\phi})^2 + (\mu^2 - 1)(e^{\lambda\phi} - 1)^2 \right] \times \\ & \left. {}_2F_1 \left[D-1, \frac{1}{2}D(1-\mu), D, 1 - e^{\lambda\phi} \right] \right), \end{aligned}$$

where $\lambda^2 = 2/((D-2)(1-\mu^2))$. We find that analytical charged solution also exists provided that $Z(\phi)$ is given by

$$Z^{-1} = \gamma e^{-\lambda\mu\phi} \left(2(D-4)(1-\mu^2) + (1-\mu)(D-2-(D-4)\mu)e^{-\lambda\phi} + (1+\mu)(D-2+(D-4)\mu)e^{\lambda\phi} \right). \quad (89)$$

The dilaton and the metric function f are then given by

$$\begin{aligned} \lambda\phi &= \log \left(1 + \frac{q}{r} \right), \\ f &= r^{1+\mu}(r+q)^{1-\mu} \left(g^2 - \frac{\alpha q^{D-1}}{r^{D-1}} {}_2F_1 \left[D-1, \frac{1}{2}D(1-\mu), n, -\frac{q}{r} \right] \right. \\ &\quad \left. + \frac{2\gamma Q^2}{(D-2)(r^{1+\mu}(r+q)^{1-\mu})^{D-3}} \right), \end{aligned} \quad (90)$$

From the expression for the field strength

$$F = -\frac{Q}{Z(\phi)r^{D-2}} \left(1 + \frac{q}{r} \right)^{\frac{1}{2}(D-2)(\mu-1)} dr \wedge dt, \quad (91)$$

we obtain the gauge potential $A = a(r)dt$, where

$$a = \frac{2\gamma Q}{r^{DD-2}} (2r + (1+\mu)q) \left(1 + \frac{q}{r} \right)^{\frac{1}{2}((D-4)\mu+2-D)}. \quad (92)$$

The neutral solution with $\mu \neq 0$ was obtained in [17] in a different coordinate system. The neutral $\mu = 0$ solution was given in [26] using the same coordinates. It is easy to calculate that the mass of the solution is given by

$$M = \frac{\alpha(D-2)q^{D-1}}{16\pi}. \quad (93)$$

It is straightforward to verify that the first law of the thermodynamics $dM = TdS + \Phi_e dQ_e$ holds, where T, S can be calculated with the standard technique and

$$Q_e = \frac{Q}{16\pi}\Sigma, \quad \Phi_e = a(r_0), \quad (94)$$

with r_0 being the location of the horizon. These thermodynamics quantities satisfy the generalized Smarr relation

$$M = \frac{D-2}{D-1}(TS + \Phi_e Q_e). \quad (95)$$

Note that this is not the usual Smarr relation (68) which breaks down in all AdS black holes unless one treats the cosmological constant as a thermodynamical variable. This generalized Smarr relation is a consequence of the scaling symmetry associated with planar AdS black holes [38]. It was shown in [38] that this generalized Smarr formula is directly related to the the universal bound [39, 40] of the viscosity to the entropy ratio in the AdS/CFT correspondence.

5.2 Class 2

The class-2 solutions are specified by the metric ansatz

$$ds^2 = -\sigma^2 f dt^2 + \frac{dr^2}{f} + r^2 dx_i^2, \quad (96)$$

The neutral scalar hairy black holes were constructed in [26], with the scalar potential given by

$$V = -2g^2(D-2)\mu(\nu - \lambda^2\phi^2)e^{2\lambda^2\phi^2} - (D-1)(D-2)\alpha e^{\lambda^2\phi^2} \left(\phi^{2\nu} + \lambda^{-2\nu}(\nu - \lambda^2\phi^2)e^{\lambda^2\phi^2}(\Gamma(\nu, \lambda^2\phi^2) - \Gamma(\nu)) \right), \quad (97)$$

where $\lambda = \sqrt{\frac{\mu}{4(n-2)}}$, $\nu = \frac{n-1}{2\mu}$. The charged planar black hole can also be constructed analytically provided that Z is given by

$$Z^{-1} = \gamma(\phi^2 + \frac{2(n-3)(n-2)}{\mu^2})e^{2\lambda^2\phi^2}, \quad (98)$$

The solution is then given by

$$\begin{aligned} \phi &= \left(\frac{q}{r}\right)^\mu, \quad F = \frac{Qe^{-\lambda^2\phi^2}}{2Zr^{D-2}}dr \wedge dt, \quad \sigma = e^{-\lambda^2\phi^2}, \\ f &= e^{2\lambda^2\phi^2}r^2 \left(g^2 + \alpha \nu \lambda^{-2\nu}(\Gamma(\nu, \lambda^2\phi^2) - \Gamma(\nu)) + \frac{\gamma Q^2}{\mu^2 r^{2(D-2)}} \right). \end{aligned} \quad (99)$$

It is straightforward to verify that the first law of thermodynamics and the generalized Smarr relation (95) both hold.

6 Conclusions

In this paper, we considered a class of Einstein-Maxwell-Dilaton theories where the dilaton coupling $Z(\phi)$ to the Maxwell field is not the usual exponential function, as in a typical supergravity theory. There are three criteria for us to determine the function Z . The first is that it should provide a toy model for string loop effects so that the extremal solution has the smooth $\text{AdS} \times \text{Sphere}$ near-horizon geometry. The second is that the theory admits two different black holes for given mass and charges, providing a counter example of the no-hair theorem. Both of the above criteria can be satisfied when $Z(\phi)$ has a stationary point. The third is that we can construct exact solutions for these black holes.

The theories we constructed indeed admit in general two types of black holes. One is the usual RN black hole, for which the dilaton scalar is a fixed constant. For the same mass and charge, there can exist also another charged black hole, in which the dilaton field varies. We analysed the global properties and obtained the first law of black hole thermodynamics, and compared the differences between the new black hole with the RN solution. We found that some scalar potential could be introduced and exact solutions of charged AdS black holes were obtained. We generalized the black holes to black p -branes.

We further obtained a class of new charged AdS planar black holes and show that they all satisfy the generalized Smarr relation, which can be viewed as the bulk dual of the viscosity/entropy ratio of the boundary field theory.

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References

- [1] G.W. Gibbons and K.i. Maeda, *Black Holes and membranes in higher dimensional theories with dilaton fields*, Nucl. Phys. B **298**, 741 (1988).
- [2] M.J. Duff and J.X. Lu, *String/five-brane duality, loop expansions and the cosmological constant*, Nucl. Phys. B **357**, 534 (1991).

- [3] A. Sagnotti, *A note on the Green-Schwarz mechanism in open string theories*, Phys. Lett. B **294**, 196 (1992) [hep-th/9210127].
- [4] J. Erler, *Anomaly cancellation in six-dimensions*, J. Math. Phys. **35**, 1819 (1994) [hep-th/9304104].
- [5] M.J. Duff, H. Lü and C.N. Pope, *Heterotic phase transitions and singularities of the gauge dyonic string*, Phys. Lett. B **378**, 101 (1996) [hep-th/9603037].
- [6] M.J. Duff, R. Minasian and E. Witten, *Evidence for heterotic/heterotic duality*, Nucl. Phys. B **465**, 413 (1996) [hep-th/9601036].
- [7] G. Aldazabal, A. Font, L.E. Ibanez and F. Quevedo, *Heterotic/heterotic duality in $D = 6$, $D = 4$* , Phys. Lett. B **380**, 33 (1996) [hep-th/9602097].
- [8] D.R. Morrison and C. Vafa, *Compactifications of F theory on Calabi-Yau threefolds. 1*, Nucl. Phys. B **473**, 74 (1996) [hep-th/9602114].
- [9] J.M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, Int. J. Theor. Phys. **38**, 1113 (1999) [Adv. Theor. Math. Phys. **2**, 231 (1998)] [hep-th/9711200].
- [10] H. Lü, S. Mukherji, C.N. Pope and J. Rahmfeld, *Loop corrected entropy of near extremal dilatonic p -branes*, Phys. Lett. B **389**, 248 (1996) [hep-th/9604127].
- [11] C. Martinez and J. Zanelli, *Conformally dressed black hole in $(2+1)$ -dimensions*, Phys. Rev. D **54**, 3830 (1996) [gr-qc/9604021].
- [12] M. Nadalini, L. Vanzo and S. Zerbini, *Thermodynamical properties of hairy black holes in n spacetimes dimensions*, Phys. Rev. D **77**, 024047 (2008) [arXiv:0710.2474 [hep-th]].
- [13] A. Anabalón, *Exact black holes and universality in the backreaction of non-linear sigma models with a potential in $(A)dS_4$* , JHEP **1206**, 127 (2012) [arXiv:1204.2720 [hep-th]].
- [14] A. Anabalón, *Exact hairy black holes*, Springer Proc. Phys. **157**, 3 (2014) [arXiv:1211.2765 [gr-qc]].
- [15] P.A. González, E. Papantonopoulos, J. Saavedra and Y. Vásquez, *Four-dimensional asymptotically AdS black holes with scalar Hair*, JHEP **1312**, 021 (2013) [arXiv:1309.2161 [gr-qc]].

- [16] A. Anabalon and D. Astefanesei, *On attractor mechanism of AdS_4 black holes*, Phys. Lett. B **727**, 568 (2013) [arXiv:1309.5863 [hep-th]].
- [17] A. Acena, A. Anabalon, D. Astefanesei and R. Mann, *Hairy planar black holes in higher dimensions*, JHEP **1401**, 153 (2014) [arXiv:1311.6065 [hep-th]].
- [18] X.H. Feng, H. Lü and Q. Wen, *Scalar hairy black holes in general dimensions*, Phys. Rev. D **89**, no. 4, 044014 (2014) [arXiv:1312.5374 [hep-th]].
- [19] C.A.R. Herdeiro and E. Radu, *Kerr black holes with scalar hair*, Phys. Rev. Lett. **112**, 221101 (2014) [arXiv:1403.2757 [gr-qc]].
- [20] P.A. Gonzalez, E. Papantonopoulos, J. Saavedra and Y. Vasquez, *Extremal hairy black holes*, JHEP **1411**, 011 (2014) [arXiv:1408.7009 [gr-qc]].
- [21] D.C. Zou, Y. Liu, B. Wang and W. Xu, *Thermodynamics of rotating black holes with scalar hair in three dimensions*, Phys. Rev. D **90**, no. 10, 104035 (2014).
- [22] Q. Wen, *Strategy to construct exact solutions in Einstein-scalar gravities*, arXiv:1501.02829 [hep-th].
- [23] C. Herdeiro and E. Radu, *Construction and physical properties of Kerr black holes with scalar hair*, Class. Quant. Grav. **32**, no. 14, 144001 (2015) [arXiv:1501.04319 [gr-qc]].
- [24] C.A.R. Herdeiro and E. Radu, *Asymptotically flat black holes with scalar hair: a review*, Int. J. Mod. Phys. D **24**, 1542014 (2015) [arXiv:1504.08209 [gr-qc]].
- [25] F. Faedo, D. Klemm and M. Nozawa, *Hairy black holes in $\mathcal{N} = 2$ gauged supergravity*, arXiv:1505.02986 [hep-th].
- [26] Z.Y. Fan and H. Lü, *Static and dynamic hairy planar black holes*, arXiv:1505.03557 [hep-th].
- [27] H.S. Liu and H. Lü, *Scalar charges in asymptotic AdS geometries*, Phys. Lett. B **730**, 267 (2014) [arXiv:1401.0010 [hep-th]].
- [28] H. Lü, C.N. Pope and Q. Wen, *Thermodynamics of AdS black holes in Einstein-scalar gravity*, JHEP **1503**, 165 (2015) [arXiv:1408.1514 [hep-th]].
- [29] H. Lü, Y. Pang and C.N. Pope, *An ω deformation of gauged STU supergravity*, JHEP **1404**, 175 (2014) [arXiv:1402.1994 [hep-th]].

- [30] G. Dall’Agata, G. Inverso and M. Trigiante, *Evidence for a family of $SO(8)$ gauged supergravity theories*, Phys. Rev. Lett. **109**, 201301 (2012) [arXiv:1209.0760 [hep-th]].
- [31] B. de Wit and H. Nicolai, *Deformations of gauged $SO(8)$ supergravity and supergravity in eleven dimensions*, JHEP **1305**, 077 (2013) [arXiv:1302.6219 [hep-th]].
- [32] H. Lü, *Charged dilatonic ads black holes and magnetic $AdS_{D-2} \times R^2$ vacua*, JHEP **1309**, 112 (2013) [arXiv:1306.2386 [hep-th]].
- [33] H. Lü, C.N. Pope, E. Sezgin and K.S. Stelle, *Stainless super p -branes*, Nucl. Phys. B **456**, 669 (1995) [hep-th/9508042].
- [34] M. Cvetič and D. Youm, *Entropy of non-extreme charged rotating black holes in string theory*, Phys. Rev. **D54** (1996) 2612, arXiv:hep-th/9603147.
- [35] F. Larsen, *A string model of black hole microstates*, Phys. Rev. **D56** (1997) 1005, hep-th/9702153.
- [36] M.J. Duff, H. Lü and C.N. Pope, *The black branes of M theory*, Phys. Lett. B **382**, 73 (1996) [hep-th/9604052].
- [37] M. Cvetič and A.A. Tseytlin, *Nonextreme black holes from nonextreme intersecting M -branes*, Nucl. Phys. B **478**, 181 (1996) [hep-th/9606033].
- [38] H.S. Liu, H. Lü and C. N. Pope, *Generalised Smarr formula and the viscosity bound for Einstein-Maxwell-Dilaton black holes*, arXiv:1507.02294 [hep-th].
- [39] G. Policastro, D.T. Son and A. O. Starinets, *The shear viscosity of strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma*, Phys. Rev. Lett. **87**, 081601 (2001), hep-th/0104066.
- [40] P. Kovtun, D.T. Son and A.O. Starinets, *Holography and hydrodynamics: diffusion on stretched horizons*, JHEP **0310**, 064 (2003), hep-th/0309213.